Initial Relative-Orbit Determination via RF Signal Localization: Part 1 of 3

Sophia Sunkin

Space Phillips

Senior at Mission Viejo High School

Dr. Alan Lovell

BACKGROUND

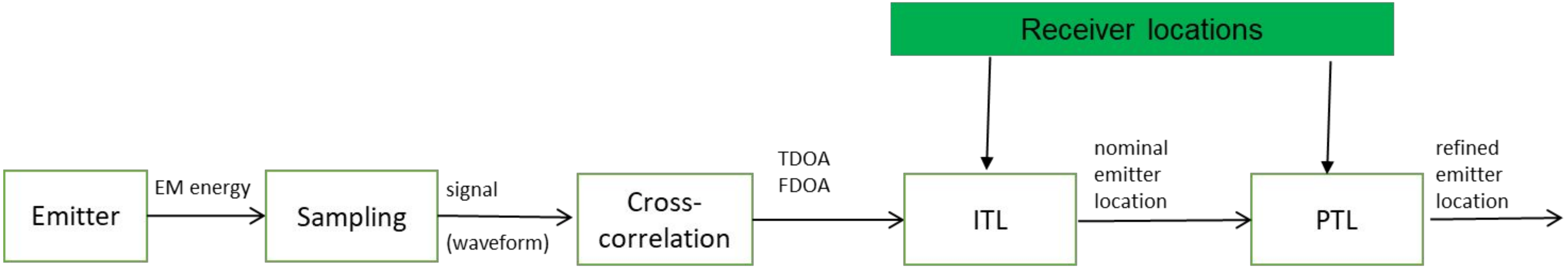
- **RESEARCH GOAL:** to improve the state of the art in precise localization of electromagnetic transmitting sources in Earth-orbiting space, from receivers that are also in orbit

- The focus of this work could best be described as passive or uncooperative localization, whereby there is no coordination between the transmitter(s) and receiver(s)

- Localization is performed based only on knowledge of the received signal itself (e.g. amplitude, phase, and frequency)

END-TO-END PROCESS

- **The common “2-step” localization process is depicted here:**
  - SIGNAL PROCESSING: cross-correlation to obtain TDOA or FDOA
  - ESTIMATION: solving measurement equations for unknown transmitter position coordinates
- **Estimation steps consist of:**
  - INITIAL TRANSMITTER LOCALIZATION (ITL): obtaining a quick/reasonable solution from minimal measurements → n equations in n unknowns
  - PRECISE TRANSMITTER LOCALIZATION (PTL): refining the ITL solution with subsequent measurements → statistical process e.g. Kalman filter



“DISTRIBUTION C – Distribution authorized to U.S. Government Agencies and their contractors; Technical content. August, 2020.”

LOCALIZATION SCENARIOS

- How do we define scenario(s) of interest?
- There are many facets to a scenario which need to be specified → we think of them as “knobs” each with different “settings”
- For this research, we’re focusing on:
- The “ITL” problem
- Utilizing two receivers
- Processing time difference of arrival (TDOA)
- Both the transmitter & receivers are in low Earth orbit (LEO) → “space-to-space”

 # OF RECEIVERS:
1
2
3
etc

MEASUREMENT TYPES:
TDOA
FDOA/FROA
SNR/phase (AOA)

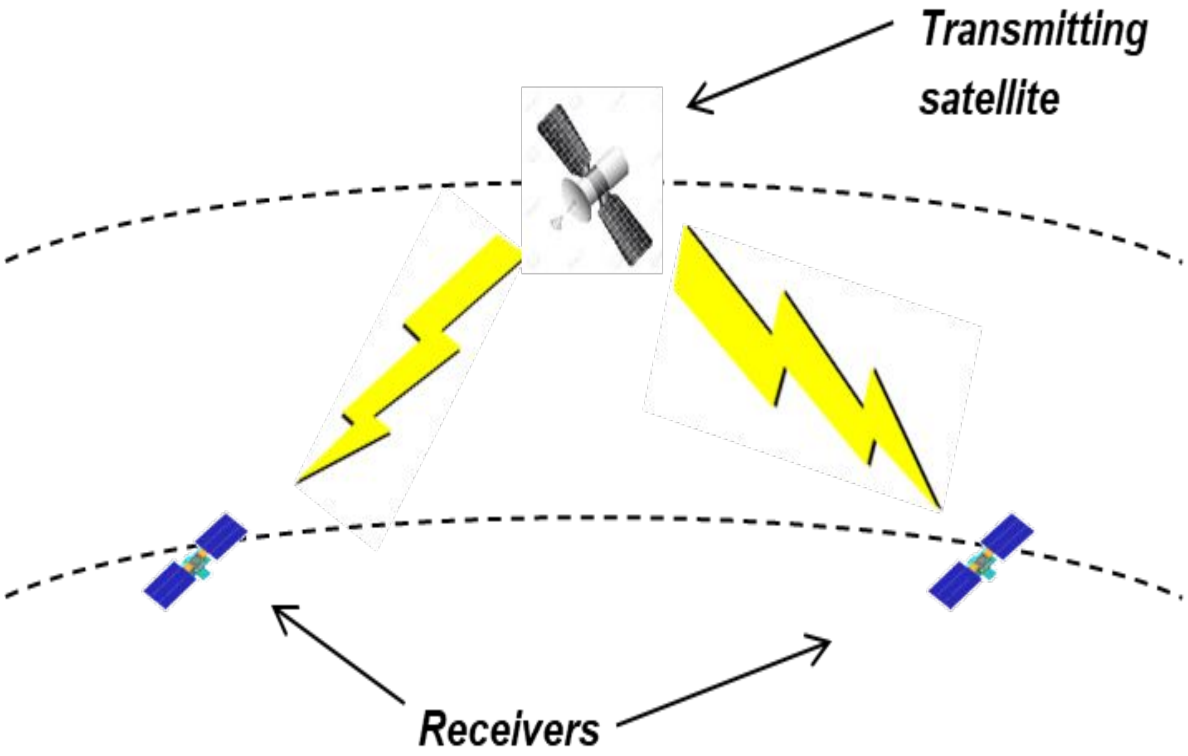
INITIAL VS PRECISE LOCALIZATION:
ITL
PTL

RECEIVER REGIME:
Terrestrial
LEO
Beyond LEO (e.g. MEO/GEO)
Hybrid (terrestrial & spaceborne)

transmitter REGIME:
Terrestrial (“geolocation”)
LEO
Beyond LEO (e.g. MEO/GEO)

LOCALIZATION SCENARIOS

- A typical space-to-space scenario might look like this:



LITERATURE

- **K. LeGrand, K. DeMars, and J. Darling, "Solutions of Multivariate Polynomial Systems Using Macaulay Resultant Expressions", *AAS***
  - Main idea: Macaulay's method is used to solve for roots of systems of multivariate polynomials
    - Finding intersections of multiple conic or quadratic functions is a fundamental problem in aerospace engineering for application such as determining orbits and/or location of objects.
    - This geometric problem is solved by finding solutions to a set of multivariate polynomials.
  - MATLAB software can be utilized to construct the M matrix and the resultant eigenvalues and null space as solutions for the variables in the set of polynomials.
- Based on Bezout's theorem the number of solutions to a set of polynomials is A^B, where A is the order of the polynomials and B is the number of equations/variables.

LITERATURE (CONT.)

- **A. Sinclair, T. Lovell, and J. Darling, “RF Localization Solution using Heterogeneous TDOA”, *IEEE***
  - Main idea: Localization of a stationary radio frequency (RF) transmitter using time-difference-of-arrival (TDOA) from pairs of receivers
  - The possible location of the transmitter falls on a hyperboloid in which the receivers are the foci.
  - The intersection of the hyperboloids created by the pairs of receivers is the transmitter location.
  - The transmitter position vector with 3 variables, x, y, and z, can be solved through a system of 3 polynomials (three hyperboloids) using Macaulay’s method.

“DISTRIBUTION C – Distribution authorized to U.S. Government Agencies and their contractors; Technical content. August, 2020.”

LITERATURE (CONT.)

- **S. Shuster, A. Sinclair, and T. Lovell, "Initial Relative-Orbit Determination Using Heterogeneous TDOA",** ***IEEE***
  - Main idea: Determining orbit of a space-based RF transmitter defined by a six element state vector [x, y, z, xdot, ydot, zdot]
  - This problem can be solved by constructing six polynomials using one of the following configurations:
    - 4 receivers - 3 TDOA measurements at 2 time points
    - 3 receivers - 2 TDOA measurements at 3 time points
    - 2 receivers - 1 TDOA measurements at 6 time points
  - The Clohessy-Wiltshire equations are used to map the various position vectors at t_k to the state vector at t_0. After substitution, the initial state vector can be solved using Macaulay's method.

SETTING UP A SCENARIO

- **For a space-to-space scenario, one of the receivers is chosen as a "reference" satellite, & each RDOA equation is written in this receiver's relative (LVLH) frame → WHY?? Because there exist several closed-form solutions for relative orbital motion, namely the linear (C-W) solution**

- **Our problem is then one of initial relative orbit determination (IROD)**

- **If we choose Receiver 1 as the reference, then the inputs/knowns are:**
  - Receiver 2 location (relative to Receiver 1) at each measurement time: x_{21}, y_{21}, z_{21}, x_{22}, y_{22}, z_{22}, … x_{26}, y_{26}, z_{26}
  - Range difference of arrival (RDOA) values at each measurement time: ρ_1, ρ_2, ρ_3, ρ_4 , ρ_5 , ρ_6
  - Note that Receiver 1 location (relative to Receiver 1) at each measurement time (x_{11}, y_{11}, z_{11}, etc) is ZERO

SETTING UP A SCENARIO

- Each RDOA equation is written as follows:

$$\rho_i = \sqrt{(x_{Ti} - x_{2i})^2 + (y_{Ti} - y_{2i})^2 + (z_{Ti} - z_{2i})^2} - \sqrt{(x_{Ti} - \overset{0}{x_{1i}})^2 + (y_{Ti} - \overset{0}{y_{1i}})^2 + (z_{Ti} - \overset{0}{z_{1i}})^2}$$

where x_{Ti}, y_{Ti}, z_{Ti} represent the transmitter's position relative to Receiver 1 at that measurement time

- C-W solution can be used to express transmitter's relative position at each measurement time t_i as a linear function of transmitter's relative states (pos/vel) at some initial (or "epoch") time t_0:

$$\begin{bmatrix} x_{Ti} \\ y_{Ti} \\ z_{Ti} \end{bmatrix} = \Phi_{pos}(t_i, t_0) \begin{bmatrix} x_{T0} \\ y_{T0} \\ z_{T0} \\ \dot{x}_{T0} \\ \dot{y}_{T0} \\ \dot{z}_{T0} \end{bmatrix}$$

3x1 3x6 6x1

SETTING UP A SCENARIO

- We know from previous work that the RDOA equation can be represented as a 2nd-order polynomial in x_{Ti}, y_{Ti}, z_{Ti}

- If we substitute $x_{Ti} = \Phi_{pos11} x_{T0} + \Phi_{pos12} y_{T0} + \Phi_{pos13} z_{T0} + \Phi_{pos14} \dot{x}_{T0} + \Phi_{pos15} \dot{y}_{T0} + \Phi_{pos16} \dot{z}_{T0}$

  and similarly for y_{Ti} & z_{Ti}, we retain the 2nd-order polynomial form, but in terms of $x_{T0}, y_{T0}, z_{T0}, \dot{x}_{T0}, \dot{y}_{T0}, \dot{z}_{T(}$ → these are the unknowns we wish to solve

- As a first step in this research, we choose scenarios in which the transmitter and receivers are in coplanar orbits → unknowns are $x_{T0}, y_{T0}, \dot{x}_{T0}, \dot{y}_{T0}$

- Each TDOA measurement results in a separate RDOA equation → 4 measurements yields a system of 4 polynomials in 4 unknowns

SETTING UP A SCENARIO

- So how do we set up a scenario to be solved?

- Start by defining:
  - Epoch time t_0 & 4 measurement times t_i
  - Transmitter, Receiver 1, & Receiver 2 orbit conditions at t_0

- Propagate all 3 satellites forward from t_0 to each t_i:

- Insert each x_{Ti}, y_{Ti}, z_{Ti}, x_{2i}, y_{2i}, z_{2i} into each RDOA equation to yield each ρ_ι

- This yields all the input values to the problem (all coefficient values of the polynomials)

SOLVING A SCENARIO

- So how do we solve a scenario?

- Bezout's Theorem states that there are 16 (finite) solutions to the polynomials

- To solve these coupled multivariate polynomials, we employ an early 20th-century linear algebra-based technique developed by Macaulay (implemented in MATLAB)

DISAMBIGUATING A SCENARIO

- Of the 16 solutions that result in a given scenario, how do we select the right one?

- Disambiguation of the solution set involves various considerations to eliminate impractical (“non-participating”) solutions
  - Eliminate extraneous solutions → solutions that don’t solve the 4 original RDOA equations
  - Retain only real (or near-real) solutions → eliminate solutions with large imaginary components

- A key fact of this research is investigating how much ambiguity exists in the problem → after performing the disambiguation steps above, how many viable transmitter orbits remain?

ISEF

- **International Science and Engineering Fair**
  - ISEF is a pre-college science research fair organized by Society for Science & the Public.
  - Every year, about 1,800 students earn the right to compete for awards in May at the Regeneron International Science and Engineering Fair (ISEF).

- This project is planned to be entered in ISEF this coming spring.

Acknowledgements

Thank you to Dr. Alan Lovell for his mentorship and tremendous support, Professor Troy Henderson and Ms. Yasmeen Hack for sharing their knowledge and support, Mr. Creighton Edington for guiding me through the ISEF application process, and Ms. Julie McCullough for facilitating my internship.